Int. J. Heat Mass Transfer. Vol. 6, pp. 387–395. Pergamon Press 1963. Printed in Great Britain.

HEAT TRANSFER IN TUBE COILS WITH LAMINAR AND TURBULENT FLOW

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(*Received* 12 *July* 1962 *and in revised form* 15 *September* 1962)

Abstract—Friction and heat transfer results are presented for the laminar flow of oil and the turbulent flow of water in tube coils having ratios of coil to tube diameter of 17 and 104, for Reynolds numbers from 12 to 65 000. A correlation for the asymptotic heat transfer coefficient for laminar flow is supported by the indication of theory, and the coefficients for turbulent flow depend on the coil diameter in the way indicated by available results for air.

NOMENCLATURE

d, tube inside diameter;
D_H, coil diameter to tube center;
f, Weisbach friction factor;
f_s, straight tube;
h, heat transfer coefficient;
k, thermal conductivity;
U_m, mean velocity;
x_1, distance along tube;
α, thermal diffusivity;
ν, kinematic viscosity.

INTRODUCTION

COMPARED to the relatively simple case of steady flow in a straight tube, the flow in a curved tube is so exceptionally complicated that even the details of the mean flow are not yet known completely. Friction factors have been determined for both laminar and turbulent flow, and recently Ito [1] has made additional contributions and has summarized well the entire status of friction in curved tubes. Long ago Adler [2] considered analytically from the boundary layer point of view the laminar flow in such tubes and determined experimentally for laminar and for turbulent flow the distribution of the mean velocity in the direction of the primary flow. Difficulties of measurement have so far played against the definition of the precise nature of the secondary flow, outward in the center section of the tube and toward the center of curvature along its walls, though sketches of the double spiral secondary flow in a curved tube do appear in connection with all treatments of the problem.

Much less is known about heat transfer in such a system. McAdams [3] cites the results of Jeschke for the turbulent flow of air in curved tubes, on the exterior of which the temperature was approximately uniform and for which the average heat transfer coefficient exceeded that for a straight tube at the same Reynolds number by the factor $(1 + 3{\cdot}5\ d/D_H)$. Little can be done analytically because of the difficulty of assessing the effects of the distortion of the mean velocity profile and because of the quantitatively unknown nature of the secondary flow, while added to the basic problem is the marked asymmetry of the flow that is demonstrated by Adler's experimental results. In this situation additional experimental data is clearly necessary to properly define the heat transfer in curved tubes and the present contribution to this consists of results on friction and heat transfer coefficients obtained in laminar and turbulent flow from two different tube coils, having ratios of coil to tube diameter of 17 and 104. These tubes were heated by electrical dissipation in the tube wall and with this approximation to constant heat flux to the fluid, local heat transfer coefficients were obtained along the length of the tube wall and also around its periphery.

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APPARATUS

Fig. 1 shows the geometry of the two coils and indicates the position of the instrumentation on them. The coils were formed from initially straight tubing of Type 321 stainless steel, 0·290 inches in inside diameter with a wall thickness of 0·012 in. Fine sand filled the tubes before bending on a mandrel to give helix diameters of 4·93 and 30·1 in respectively. Deformation occurred in bending so that for the small helix the major axis of the resulting elliptical section, which was perpendicular to the helix radius, was 5 per cent greater than the minor axis; for the large coil, this difference was 1 per cent. Buckling occurred on the inner surface and this was evident in corrugations with a wave height of 0·001 in. The tubes were 104 and 107 in long, providing for the large coil one complete turn with 5-in tangential inlet and outlet sections, and for the small coil 6½ turns with 3-in tangential sections.

The flow system consisted of the coil, pump, flow metering, and auxiliary heating components. Heating of the coil was accomplished by alternating current which entered the coil through bus

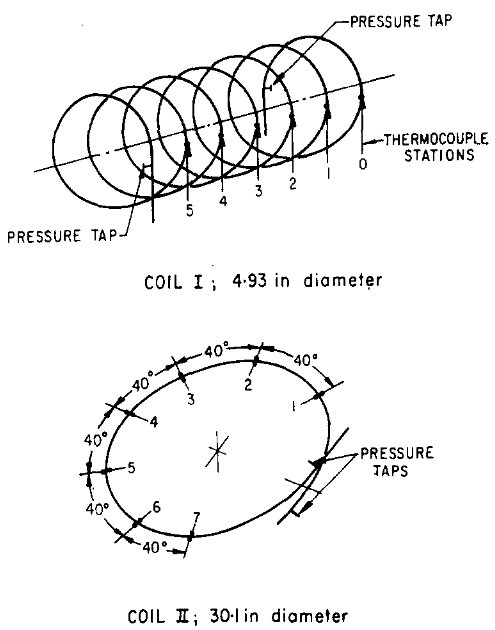

FIG. 1. Geometry of the two coils. The numbers indicate locations at which thermocouples were affixed to the tube surface on the inside and outside of the coil.

connections at its ends, and external thermal insulation was provided by Santocel filling in the box in which the coil was contained.

Pressure taps were placed in the end sections and mixing cups were provided in the end connections. Thermocouples were located in these cups and also at a number of points along the tube, on the outer and inner surfaces of the helix, as indicated on Fig. 1. In addition, at station 5 on the small coil, additional thermocouples were placed at 36° intervals between the inner and outer thermocouples, and on the large coil such additional thermocouples were placed at stations 1, 4, and 7. All tube thermocouples were 40 gage iron–constantan, separated from the tube by one-half mil thick Mylar tape and held to the tube by a wrap of pressure sensitive tape of the same material.

Apparent heat transfer coefficients were based on the electrical power input, assumed to be dissipated uniformly per unit volume of the assumed annular tube wall cross section. The temperature of the inner surface of the tube wall was inferred from the measurement of the temperature of its outer surface in conjunction with the analytic solution for a hollow cylinder in which heat is generated uniformly. The fluid temperature was obtained from the linear variation of mean fluid temperature between the inlet and outlet mixing cups, consequent to the uniform input of heat along the length of the tube.

Clearly, in the heat transfer coefficients so evaluated there are neglected changes in local resistivity due to variable cold working, changes in geometry, and all circumferential conduction of heat, and of these, the first two have been estimated to introduce into the heat transfer coefficient errors no greater, and probably much less, than 10 per cent. The last effect is large in some cases and partial evaluation of the "true" coefficient, in which such conduction is accounted for, was made with the information from those stations at which were situated additional circumferential thermocouples. The difference between the "apparent" and "true" coefficients is substantial in laminar flow when the heat transfer coefficients are small; in turbulent flow, the heat transfer coefficients were high enough so that, in the system concerned, the peripheral conduction was negligible.

RESULTS WITH LAMINAR FLOW

Laminar flows were obtained by the use of a medium heavy Freezene oil, with operating conditions in the following range:

Flow rate	22–1500 lb/h
Heat flux	146–6600 Btu/h ft^2
Fluid temperature	96–193°F
Wall-to-fluid temperature difference	2–56°F
Viscosity ratio, wall-to-mean fluid	1·05–3·6
Minimum heat transfer coefficient	20–580 Btu/h ft^2 degF
Minimum Nusselt number	6–200
Reynolds number	12–5600
Prandtl number	100–657.

Fluid properties were always evaluated at a film temperature which was the average of the mixed mean fluid and the tube inner wall temperatures.

The friction factor was deduced from the overall pressure drop and was evaluated on the basis of fluid properties at the mean film temperature. Most results for friction were obtained from the small coil and these results are pictured on Fig. 2 as a function of the Reynolds number for both isothermal and non-isothermal flow. These friction factors are about 8 per cent below White's [4] formulation:

$$\frac{f}{f_s} = \left\{1 - \left[1 - \left(\frac{11{\cdot}6}{(U_m d/\nu)\sqrt{(d/D_H)}}\right)^{0{\cdot}45}\right]^{2{\cdot}22}\right\}^{-1}$$

$$11{\cdot}6 < \frac{U_m d}{\nu}\sqrt{\left(\frac{d}{D_H}\right)} < 2000, \quad f_s = \frac{64}{U_m d/\nu}. \quad (1)$$

The friction factors obtained from the large coil scattered a little more than those for the small coil which are shown on Fig. 2, but the factors for the large coil are predictable, on the average, from the White equation. Thus a representation of the ratio f/f_s as a function of the Dean number $(U_m d/\nu)\sqrt{(d/D_H)}$ will, in the applicable range, show the friction factors for both coils to be within 8 per cent of the White equation.

Fig. 3 shows results for the apparent heat transfer coefficient, for the inside and the outside of the small coil, in the form usual for heat transfer to laminar flow in a straight pipe and there is shown for this case the prediction of Siegel [5] for a fully developed laminar flow in a straight pipe with constant heat flux at the wall. The heat transfer coefficients for the coil

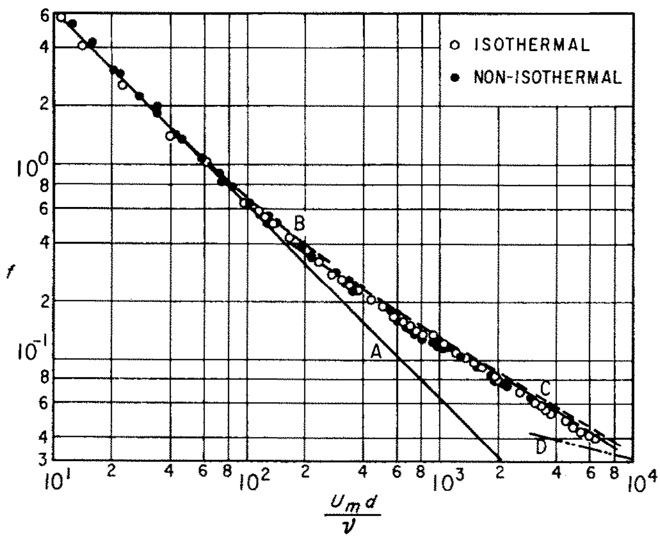

FIG. 2. Friction factors for the small coil with laminar flow. The curves indicate predictions of the friction factor: A, straight tube; B, White, equation 1; C. Prandtl, curved tube; D, turbulent flow, straight tube.

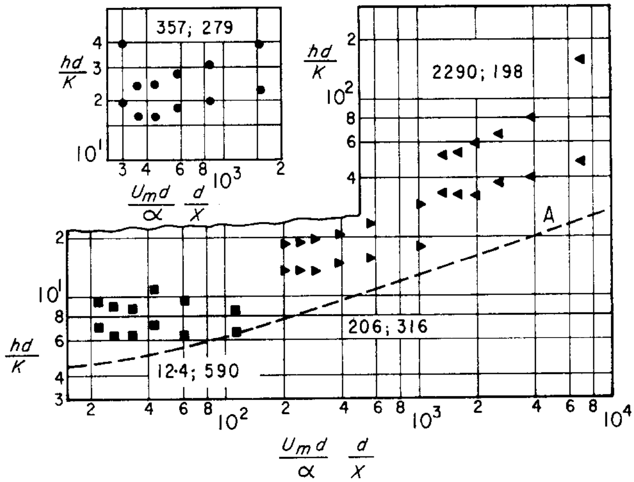

FIG. 3. Local heat transfer coefficients for laminar flow. Results obtained from the small coil at the Reynolds; Prandtl numbers indicated. The points refer to the outside and inside of the coil, the outside giving the higher coefficients.

are higher on the outer surface than on the inner surface and both exceed substantially the values that exist in a straight pipe. With large Peclet numbers there is a monotonic decrease in the coefficients as the distance downstream increases, while for intermediate and low Peclet numbers irregular behavior sometimes became apparent. One aspect thereof is the almost cyclic character shown by the results for a Reynolds number of 12·4, and another is the situation demonstrated by the inset of Fig. 3, in which the coefficients decrease normally until they rise in value at the last station. This behavior was found with both the large and small coils, and was emphasized as the difference between wall and fluid temperatures increased. This implies a free convection effect, which might be anticipated because of the substantial body forces arising from the curvilinear motion. No consistent correlation was achieved from this view, however, and the small number of runs in which this irregular behavior occurred have not been explained so far.

The salient features of the local "apparent" heat transfer coefficients as they are shown in Fig. 3 is the excess over the values expected for a straight tube and the tendency toward asymptotic values for heated lengths that are much less than those that are required in a straight tube for the same Peclet number. In regard to the excess of the values above those for flow in a straight tube with a parabolic profile, it is of interest but probably of no fundamental importance, to note that the slug flow result is almost an upper bound for the coefficients shown on Fig. 3 for the outside of the coil. But the effect of curvature is not to make the mean speed profile more uniform than the parabolic one of the straight tube, but to produce even greater distortion, with the maximum velocity near the outside of the cross section, where the maximum heat transfer coefficient indeed does occur.

The tendency toward early asymptotic behavior is a feature which is capable of some generalization and to examine this region alone the local values found in the region of reasonably invariable heat transfer coefficient in the runs pictured in Fig. 3 and from many additional runs with the small coil are shown on Fig. 4 as part of the group $hd/k\ (\nu/\alpha)^{-0\cdot 3}$. This is an empirical choice which produced the best fit of the experimental results, for Prandtl numbers from 100 to 650. Clearly, the correlation of these "asymptotic" values is inextensible to low Prandtl numbers, for then the correlation would indicate Nusselt numbers less than 4·36, the asymptotic value for a straight pipe.

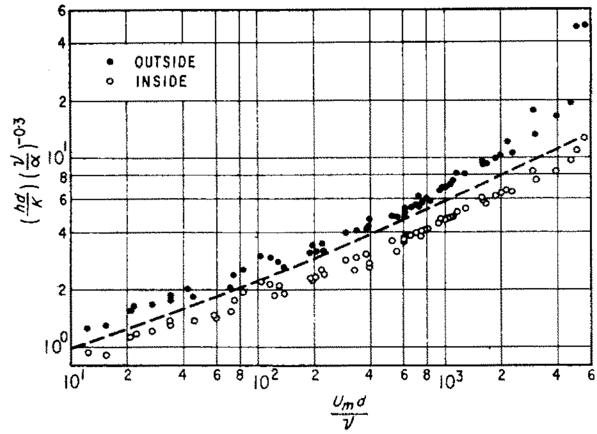

FIG. 4. Asymptotic coefficients for the small coil. The curve is equation (3) with A = 0·13.

The "apparent" coefficients for the inside and the outside of the coil are, as shown on Fig. 4, in the ratio of about 1·5. True values could be obtained at the last station of the test section at which were located additional peripheral thermocouples, from the indication of which the circumferential heat flux could be determined and the true peripheral flux to the fluid thus determined. Fig. 5 shows typical results that were found in this way and the left side of this figure illustrates the usual case of regular behavior with the maximum coefficient existing at the outer edge of the coil. In terms of these true coefficients, the ratio of the outside to the inside coefficients is four.

Dashed lines on Fig. 5 show the magnitude of the average heat transfer coefficient, obtained from the total heat generation over the periphery and the average temperature over the periphery.

Fig. 5 shows also, on its right half, the distorted peripheral distribution that existed in the anomalous runs for which asymptotic behavior was not obtained. While the minimum heat transfer coefficient still existed at the inner surface of the coil, the maximum coefficient in some cases shifted to 90° from the inner edge. The location at which this distribution existed was, of course, beyond the point at which the minimum heat transfer coefficients occurred.

Fig. 6 shows the asymptotic values obtained with the large coil, presented in the same way as are those on Fig. 4 for the small coil. The scatter of points on Fig. 6 is somewhat greater and while the results for the inside of the coil correspond generally with those for the small coil, the values for the outside are lower, particularly for small Reynolds numbers. A general comparison between the two figures reveals little effect of coil diameter.

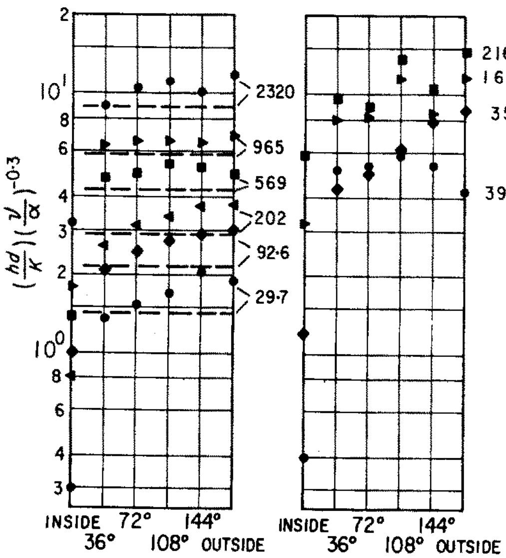

Fig. 5. Peripheral variation of the heat transfer coefficient. Reynolds numbers are indicated for each run. The results on the right are associated with anomalous behavior.

The correlation of the asymptotic values of the heat transfer coefficient that is achieved on Figs. 4 and 6 suggests that a relation of these heat transfer coefficients to the friction factor

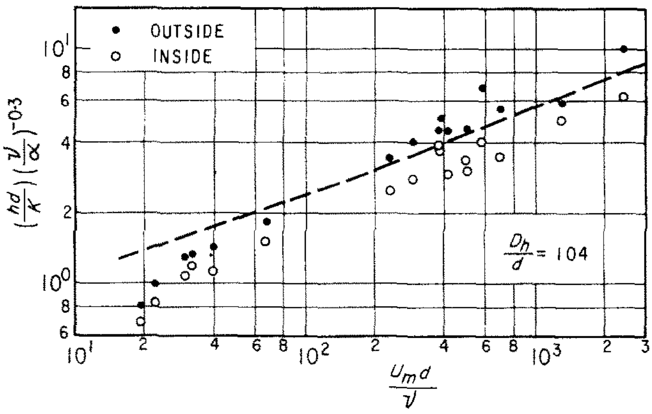

FIG. 6. Asymptotic coefficients for the large coil. The curve is equation (3) with $A = 0.13$.

should be obtainable. The Leveque result for a flat plate is a reasonable approximation for the initial part of the thermal entry length in a straight tube and has been classically so used. But now if it is to be applied to the asymptotic values under consideration some kind of continuous "thermal entry" phenomenon must be visualized and such a situation does exist if the secondary flow is considered as the preponderant transfer mechanism in the asymptotic region. In this, the fluid might be considered to emerge from the central region of the flow at the outside of the flow cross section, and pass along the tube wall as it gradually traverses the semi-circumference to the inner edge of the flow cross section, where it again re-enters the interior of the flow, to emerge again on the outer side. Combined with the Leveque solution, this picture is sufficient to specify the variation of the peripheral heat transfer coefficient but this variation is not like that shown in Fig. 5, but rather the opposite form is indicated. Ignoring this discrepancy, however, and using the average heat transfer coefficient for the length, x_1, over which the fluid has contacted the pipe wall, which is also the average at any cross section because of the different points of emergence of the fluid filaments at the various peripheral points, the average coefficient is specified as

$$\frac{hd}{k}\left(\frac{\nu}{a}\right)^{-1/3} = 0.644\left[\frac{f}{8}\left(\frac{U_m d}{\nu}\right)^2 \frac{d}{x_1}\right]^{1/3}. \quad (2)$$

If this equation is taken in the alternative form

$$\frac{hd}{k}\left(\frac{\nu}{a}\right)^{-1/3} = A\left[\frac{f}{8}\left(\frac{U_m d}{\nu}\right)^2\right]^{1/3} \quad (3)$$

and if there is ignored the small difference in the exponents of the Prandtl number as these occur in equation (3) and in the ordinate of Fig. 4, then this equation, evaluated with the friction factors that are given on Fig. 2, can be placed on Fig. 4 once a value is chosen for the factor A. The use of $A = 0.13$ orients the resulting curve in the approximate position of the average peripheral heat transfer coefficient and Fig. 4 reveals the fit to be quite good over the entire experimental range. This same curve, with $A = 0.13$, is shown also on Fig. 6, in comparison with the results for the large coil; the fit is not nearly as good, though except for low Reynolds numbers, the position of the curve is acceptable. The distance x_1/d is the remaining factor that would change A for the larger coil; Adler's [2] theory indicates that x_1/d should vary as $(D_H/d)^{1.2}$ so that the value of A for the large coil might be expected to be about $[(17/104)^{1/2}]^{1/3} = 0.74$ times that for the small coil. Such a change would improve the correspondence on Fig. 6 but it is scarcely justifiable because of the tenuous theoretical application that is involved in this whole view of the problem.

What essentially does emerge, without any specific theoretical implications, is the relative suitability of equation (3) for the specification of the minimum peripheral average heat transfer coefficient to be expected for tube coils of diameter ratios between 17 and 104, for the range of Prandtl numbers of the experiments.

RESULTS WITH TURBULENT FLOW

A consequence of tube curvature is the elevation of the Reynolds number at which transition to turbulence takes place and the diminution of the distinction between the friction factors for laminar and turbulent flow. Because of the high Reynolds numbers that are required, turbulent flow was obtained only with water, and results were obtained with both large and small coils, with operating conditions as follows:

Flow rate	230–1400 lb/h
Heat flux	300–28 100 Btu/h ft^2
Fluid temperature	80–140°F
Minimum heat transfer coefficient	427–5700 Btu/h ft^2
Minimum Nusselt number	15–240
Reynolds number	6000–65 600
Prandtl number	2·9–5·7

Friction factors for isothermal and non-isothermal conditions were obtained from both the large and small coils, and these results are shown on Fig. 7, and are compared there to experimental results of Ito [1] for coils of similar ratio of helix to tube diameter. Ito specified his results, and inferentially the present ones also, by the relation:

$$\frac{f}{f_s} = \left[\frac{U_m d}{\nu}\left(\frac{d}{D_H}\right)^2\right]^{1/20}, \quad \left(\frac{U_m d}{\nu}\right)\left(\frac{d}{D_H}\right)^2 > 6. \quad (4)$$

Because of the reduced effect of curvature on friction in turbulent flow, this formula is only about 6 per cent low, even for values of $(U_m d/\nu)$ $(d/D_H)^2$ equal to unity, so that within this error it predicts all the turbulent friction factors measured on the two coils.

Ito specified the initial point of turbulent flow in terms of a critical Reynolds number

$$\left(\frac{U_m d}{\nu}\right)_{cr} = 2 \times 10^4 \left(\frac{d}{D_H}\right)^{0.32}. \quad (5)$$

This point is indicated on Fig. 7 for both coils. The present results for the large coil are inadequate for judgment in this regard, but those for the small coil do verify the predicted transition point.

Heat transfer coefficients were determined for water flow, and the situation regarding these coefficients is simpler than it was for laminar flow. There was no evidence of thermal entry length, this being completed before the first thermocouple station, and there was no evidence of longitudinal variation of the coefficient, as occurring in the anomalous laminar runs. Because the heat transfer coefficients were very high, the effects of circumferential conduction were negligible and the "apparent" coefficients can be taken to be the true values. The high heat

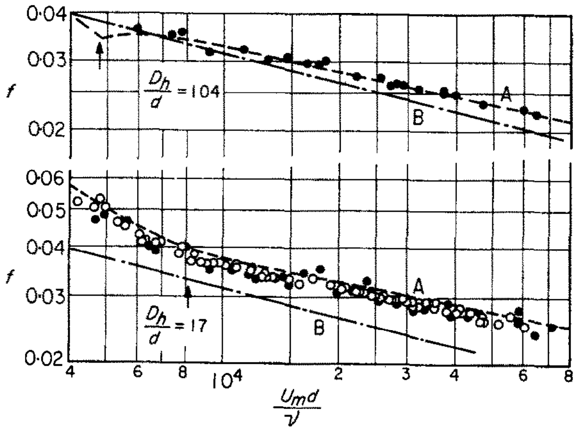

FIG. 7. Friction factors for turbulent flow. Curves A are the results of Ito for D_H/d = 16·4 and 100; curves B are the friction factor for a straight tube. The arrows indicate transition points indicated by equation (5).

transfer coefficients also made the difference between wall and fluid temperatures small, and particularly at the outside of the helix this difference became of the order of the temperature drop through the tube wall. Slight inaccuracy of measurement then produced significant scatter in the results.

Fig. 8 presents heat transfer coefficients for the large coil in terms of the group $(hd/k)\,(\nu/\alpha)^{-0\cdot4}$ as would be specified by the kind of correlation normally used for a straight pipe. As always, fluid properties are evaluated at the mean film temperature. The circumferential variation

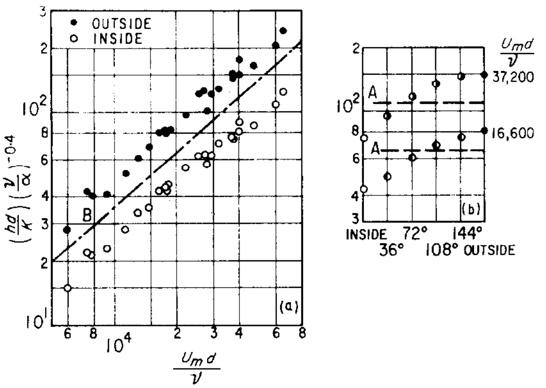

FIG. 8. Turbulent heat transfer in the large coil. Curves A indicate the circumferential average heat transfer coefficient, curve B is equation (6). Fig. 8b gives the circumferential variation at the indicated Reynolds numbers.

of the coefficient is shown for two Reynolds numbers, and these indicate that the ratio of outside to inside coefficients is of the order of 2, less than in laminar flow because of the elimination of the pronounced minimum in the inside coefficient. The magnitude of the average circumferential coefficient, defined in terms of the average temperature around the periphery, is indicated by dashed lines on Fig. 8b and the average coefficients for all the runs deviate by less than 10 per cent from curve B on Fig. 8a. Curve B, in turn, is the equation

$$\frac{hd}{k}\left(\frac{\nu}{\alpha}\right)^{-0\cdot4} = \frac{f}{8}\,\frac{U_m d}{\nu} \qquad (6)$$

evaluated with friction factors obtained from equation (4) and from the usual expression for the friction factor for a straight tube, $f_s/8 = 0{\cdot}023\,(\nu/U_m d)^{0\cdot20}$. The excellent correspondence with this analogy prediction is exceptional in view of the large peripheral variation that is included in the average value.

Less satisfactory results were obtained from the small coil, with which operating difficulties produced a reduction in the reliability of the thermocouples on the outside of the coil and this, combined with the large magnitudes of the coefficients for turbulent flow, combined to produce scatter in the results for the outside of the coil, as shown on Fig. 9a. Fig. 9b shows, for three of the more satisfactory runs, the larger peripheral variation of the local heat transfer coefficient, the ratio of the outside to the inside

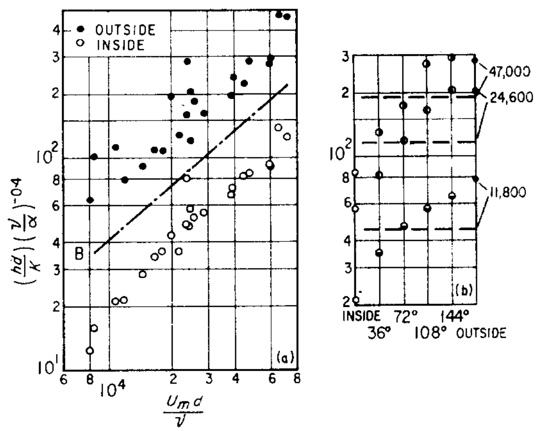

FIG. 9. Turbulent heat transfer in the small coil. The representation is the same as on Fig. 8.

values being of the order of four for the small coil. Despite this greater variation, the average coefficients are still in good accord with equation (6), though for this case the variation is of the order of 15 per cent because of the erratic nature of many of the determinations of the outside wall temperature.

For heat transfer in coils, McAdams [3] recommends Jeschke's result for air flow, in which the average heat transfer coefficient for the coil was found to exceed that for a straight pipe by the factor $[1 + 3{\cdot}5\,(d/D_H)]$. The comparison of this with (4) and (6) is as follows:

D_H/d	$[1 + 3{\cdot}5(d/D_H)]$	Equations (4) and (6) $Re = 10^4$	$Re = 10^5$
17	1·20	1·27	1·34
104	1·03	1·06	1·12

Thus, there is little choice between the two recommendations, though the results for the small coil do indicate the slightly greater Reynolds number dependence that is given by (4) and (6), rather than the continued 0·8 power relation implied by Jeschke's recommendation.

CONCLUSIONS

Results have been presented for heat transfer and friction for laminar flow of oil and turbulent flow of water in coiled tubes having ratios of coil to tube diameter of 17 and 104.

The friction factors for laminar and turbulent flow correspond with the results of Ito and are predictable by his equations when for non-isothermal flow the properties are evaluated at the mean film temperature.

In laminar flow, the heat transfer coefficients revealed an effect of thermal entry length. Except for certain anomalous runs, the entry effect terminated and asymptotic coefficients could be estimated. Average circumferential coefficients were estimated from these asymptotic values, and the Reynolds number dependence thereof corresponds to an estimate made on the basis of the Leveque theory.

In turbulent flow, the appraisal of the results for the heat transfer coefficient was simplified because the variation of the coefficient was only peripheral and not also longitudinal. Average heat transfer coefficients for the periphery can be specified accurately from the usual analogy formulation, using friction factors appropriate for curved tubes.

ACKNOWLEDGEMENT

The authors acknowledge the contribution of Mr. F. Barrera, who secured the results for turbulent flow, and to Mr. J. Ingamels for his initial examination of heat transfer coefficients in these systems.

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Résumé—Les auteurs présentent des résultats sur la transmission de chaleur et le frottement dans le cas d'écoulements laminaires d'huile et d'écoulements turbulents d'eau dans des serpentins ayant des rapports diamètre de la spirale/diamètre du tube de 17 et 104, pour des nombres de Reynolds de 12 à 65.000. A l'aide des indications données par la théorie, ils établissent une relation pour le coefficient de transmission de chaleur asymptotique dans le cas d'un écoulement laminaire. Les coefficients, dans le cas de l'écoulement turbulent, dépendent du diamètre de la spirale de la même façon que dans les résultats obtenus pour l'air.

Zusammenfassung—Für laminaire Strömung von Öl und turbulente Strömung von Wasser in Rohrspiralen mit Verhältnissen von Windungs- zu Rohrdurchmesser von 17 bis 104 und Reynoldszahlen von 12 bis 65 000 sind Reibungs- und Wärmeübergangsergebnisse angegeben. Eine Beziehung für den asymptotischen Wärmeübergangskoeffizienten für laminaire Strömung wird von der Theorie unterstützt; der Koeffizient für turbulente Strömung hängt so vom Windungsdurchmesser ab, wie es für Versuche mit Luft bereits angegeben ist.

Аннотация—Приводятся результаты экспериментального исследования трения и теплообмена в ламинарном потоке нефти и в турбулентном потоке воды в змеевиках с отношениями диаметра змеевика к диаметру трубы, равными 17 и 104, в диапазоне чисел Рейнольдса от 12 до 65000. Корреляция асимптотических значений коэффициента теплообмена подтверждается теорией. Коэффициенты теплообмена для турбулентного потока нефти зависят от диаметра змеевика точно также, как и при использовании известных результатов для воздуха.